Analysis of the Brazilian National Learning Standards for Mathematics Version 3

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Contents

High Level Feedback

The Mathematics course presents a balanced progression of learning across the years of schooling. Some previous recommendations have been taken into account in this version of the Mathematics course. However, further suggestions have been made that will provide greater internal coherence and alignment with trends in international Mathematics Curricula.

The structure of Mathematics in the BNLS compared with international trends

The proposed Mathematics course consists of five theme units across nine years of Elementary and Middle School:

- Numbers
- Geometry
- Quantities and Measurements
- Algebra
- Probability and Statistics

These themes are almost the same as the previous version. The titles have been slightly changed but the content is identical. There is no reduction in the number of themes as per the previous recommendations in relation to geometry and measurement, number and operations, algebra and functions, statistics and probability.

As stated in the earlier advice, there is an international trend towards the inclusion of a process strand in mathematics curricula and learning standards. This strand ensures a purposeful development of students' mathematical skills and competencies across content areas and through the years of schooling.

1. Recommendation:

It has been noted that the recommendation previously suggested to include a process strand has not been enacted. However, the inclusion of a process strand is still recommended to ensure the parity of the BNLS with international curricula. This process strand could underpin the five BNLS themes

Numbers	Geometry	Quantities and Measurements	Algebra	Probability and Statistics
Mathematical Processes				

Volume

There has been more content added into the BLNS by increasing the number of learning objectives in both the elementary and middle school. It is assumed that not all learning objectives require an equal amount of time for students to learn, but there has been approximately 40% more content added.

	Elementary School	Middle School	Total
BNLS	132 (previously 95)	106 (previously 75)	238 (previously 170)
British Columbia	67	59	126
Australia	100	107	207
Singapore	206	NA	

Consideration needs to be given as to the value that these new learning objectives add to the quality of the learning that students will undertake when the BNLS is introduced.

2. Recommendation:

Evaluate the amount of content that has been included in the BNLS with regard to the time allocated to teaching Mathematics within a whole learning programme.

Measurability

The majority of learning objectives in Elementary and Middle schools are measurable. Most objectives begin with an observable and measurable verb. However, there are still a number of learning objectives that may not be easily measurable. For example:

- Understand
- Recognise
- Establish

The recommendation in previous advice was to replace verbs that are difficult to measure with verbs that result in an action or a performance that can be observed or measured. There are still a number of verbs included in the current document that would be difficult to measure.

3. Recommendation:

Replace the immeasurable verbs in the BNLS with action of performance verbs that can be observed or measured.

Academic rigour

It is difficult to make judgements about the level of academic rigour of the learning objectives other than an analysis of the verbs used to describe the learning that takes place. Some of the following statements and comment may be influenced by nature of the translation. In Elementary there are 26 different verbs used at the beginning of each of the learning objectives. The most common verbs used are ‘recognise’ and ‘solve’. Similarly, these two verbs are the most commonly used amongst the 25 different verbs in the Middle school. There are only 11 common verbs amongst the total number of verbs used across Elementary and Middle School. It would be assumed that the higher order thinking verbs would occur in the Middle school as a way of developing the level of conceptual development, but this is not evident. There has been significant effort to incorporate the suggestions from previous advice but there needs to be a progression of verbs across the years of schooling to demonstrate the cognitive development of the conceptual demands of the learning objectives. In the Elementary years, it would be expected to see mainly measurable verbs such as ‘describe’ ‘identify’ ‘measure’, and in the Middle years verbs such as ‘analyse’, ‘solve’, ‘interpret’.

4. Recommendation

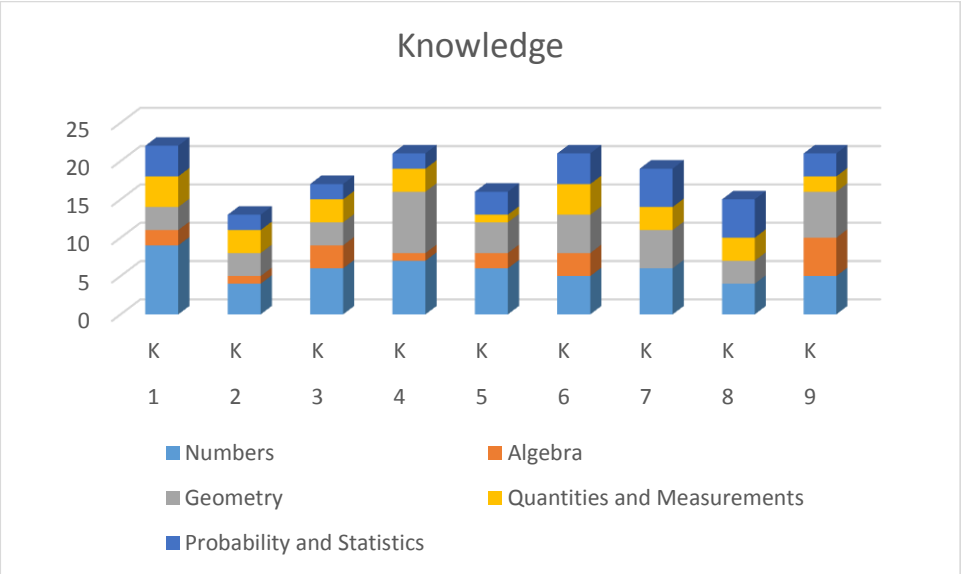
The Learning Objectives be revised to achieve a balance of verbs to demonstrate the progression of cognitive development across the years of schooling.

Balance between the stands

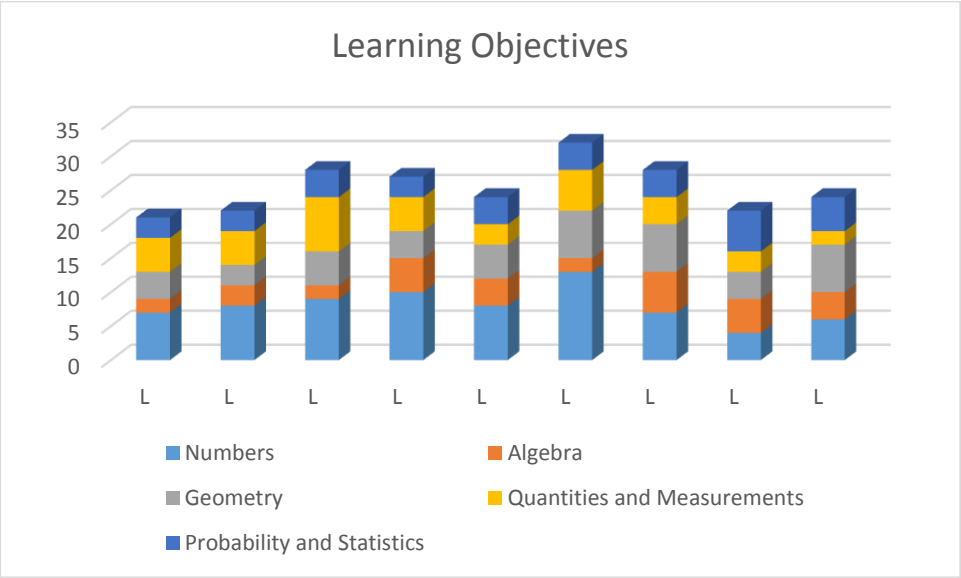
The number of Knowledge Objects and Learning Objectives in each theme unit for each year level are recorded in the table below.

	1		2		3		4		5		6		7		8		9	
	K	L	K	L	K	L	K	L	K	L	K	L	K	L	K	L	K	L
Numbers	9	7	4	8	6	9	7	10	6	8	5	13	6	7	4	4	5	6
Algebra	2	2	1	3	3	2	1	5	2	4	3	2	0	6	0	5	5	4
Geometry	3	4	3	3	3	5	8	4	4	5	5	7	5	7	3	4	6	7
Quantities and Measurements	4	5	3	5	3	8	3	5	1	3	4	6	3	4	3	3	2	2
Probability and Statistics	4	3	2	3	2	4	2	3	3	4	4	4	5	4	5	6	3	5

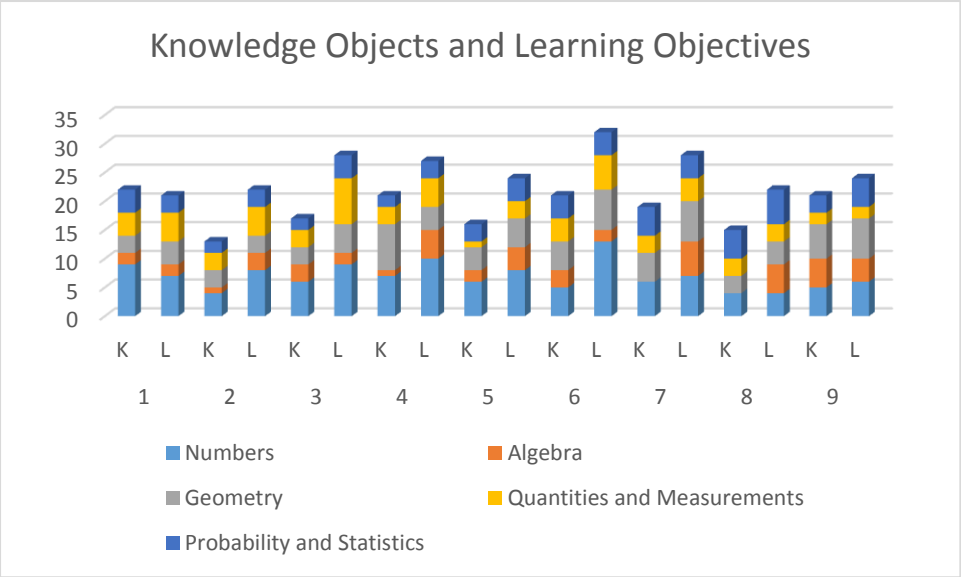
The Knowledge Objects for Numbers decrease over the years of schooling. This is a result of the abstract nature of Mathematics developing over time and students become more confident in the use of numbers. This is evident in the Knowledge graph below. As expected, Algebra becomes more prominent as the years progress. The documentation received did not contain any Knowledge Objects for Algebra for years 7 and 8. This may be an oversight in the documentation and so is not reflected in the graphs. Geometry increases after the early years and remains fairly constant across the years. Probability and Statistics are more evident in the beginning of the Middle years. In most international curricula, the amount of Algebra, Geometry, Probability and Statistics increase as students progress through the years.



There is no trend in the number of Learning Objectives relating to Numbers. This is evident in the Learning Objectives graph below. The development of Algebra skills is demonstrated in the graph below. The increase of the Algebra skills occurs in Years 7, 8 and 9. Geometry skills are more evident in the later years of schooling. Probability and Statistics has a constant number of Learning Objectives until the later years of Middle School



The combined graph below demonstrates the differences in emphases in the Knowledge Objects and Learning Objectives at year levels. Years 2 and 3 have significant differences in the number of Knowledge Objects and Learning Objectives. That could be because each Knowledge Object may have multiple learning Objectives aligned to it. Similarly, in Years 6 and 7 here are significant differences as well.



5. Recommendation:

Include the Algebra Knowledge Objects into the documentation for Years 7 and 8. Consider the alignment between the number of Knowledge Objects and the number of Learning Objectives in each year.

Clarity

The introductory sections of the document are predominantly prose with few headings or subheadings. This was noted in the previous advice. The intention of the curriculum could be made clearer with subheadings, bullet points, tables and diagrams to emphasise the key organisers of the Mathematics curriculum.

The Knowledge Objects are not consistently written. Some are long sentences beginning with a verb or participle, for example: in Year 5 'Converting between decimals and fractions: recognition, meanings, reading, representation on a number line'. Others are a single word, representing a topic, for example: in Year 5 'Percentages'. Whereas, others are a statement, representing a topic or concept, for example: also in Year 5,'Proportional variation between two quantities'.

The Knowledge Objects often contain multiple concepts within the one statement. For example: in Year 4, 'Collecting, classifying and representing data. Reading, interpreting and representing data in double-entry tables, simple and clustered column graphs, bar graphs and picture graphs'. This may cause some confusion and not clarify the intent of the Knowledge Object. If there is more than one concept it may be useful to separate them into two or more Objects.

The Learning Objectives are written more consistently. Most begin with a verb and are better described as action statements. The same verbs are used in year 1 and in Year 9. This is at times appropriate, but it would be assumed that there would be an increased number of verbs in the later years concerning 'analyse' 'interpret' etc.

6. Recommendation:

The Knowledge Objects be reviewed and revised to ensure that they are clear and consistent. Consideration should be given to grouping the Knowledge Objects into threads for consistency across the years. For example: Quantities and Measurements could have threads across all the year levels under the Knowledge Objects of 'Using measurements', 'Time', 'Shapes' and 'Money'.

Pitch, progression and sequence

The Knowledge Objects are mostly age appropriate across the years of schooling. It is assumed that students would enter Year 1 with some understanding of Numbers as the correspondence between numbers and a collection is not necessarily developed.

The alignment between the Knowledge Objects and the Learning Objectives seems to be arbitrary at times and not evident at all at others. In some cases, the Knowledge Object is more or less repeated in the Learning Objective. There is limited emphasis on the use of digital technologies.

7. Recommendation:

Align the Knowledge Objects with the Learning Objectives so there is a clear line of sight between them. Consider including more references to the use of digital technologies, especially in the themes of Quantities and Measurements and Probability and Statistics.

Analysis with Recommendations for Improvement

Subject Fundamentals

The introductory text provides a rationale for the inclusion of Mathematics in the school curriculum. It describes the reasons for having successful mathematics learners in schools including how mathematics contributes to the development of students in other learning areas.

It describes the Mathematics General Competencies for Elementary and Middle schools of Personal and Social Competencies (SOC), Cognitive Competencies (COG), and Communicative Competencies (COM). A table describes where these competencies are evident in the curriculum with nine dot points. However, this is the only place where the competencies are identified.

8. Recommendation:

Identify the General Competencies within the curriculum document.

Structure

The text describing the structure of the stands in the Mathematics curriculum has taken into account advice given in previous documentation. Each strand is described in depth and the relationship between the strands has been clearly identified.

It is noted that the number of strands has not been reduced in this version which may lead to overcrowding of the curriculum. Also, a process strand has not been developed which may assist to underpin the learning of Mathematics. This advice in the previous documentation has not been taken on board despite the trends in curricula in high performing countries.

The previous advice also suggested using the new curriculum from British Columbia as a model to include a way of combining content and competencies. The Singaporean curriculum also reflects a content and competency model. In the Australian Curriculum: Mathematics, the process strands are called the 'Proficiencies' and include; understanding, fluency, problem solving and reasoning. While not all the proficiencies apply to every content description, they indicate the breadth of mathematical actions that teachers can emphasise.

9. Recommendation:

Include process strands that underpin the learning of Mathematics across Elementary and Middle Schools.

Introductory text

The introduction describes the understanding that students will develop during Elementary and Middle Schools. The use of technology has been included in the text with reference to how it would be used in a learning context but is not then translated into the Learning Objectives

The last paragraph of the Learning Mathematics in Elementary School (4.4.1.1) discussed the need for understanding as an intrinsic aspect of the Mathematics Curriculum. The notion is aspirational but there is little evidence of 'understanding' in the Knowledge Objects. The verb, 'understand' only occurs 8 times in the skills within Elementary Mathematics and 'understanding' only occurs 7 times amongst the Learning Objectives. If being able to

understand were a priority for Elementary school, it would be expected that the terminology would be more prevalent and visible within the Learning Objectives.

10. Recommendation:

Ensure the wording of the introductory text reflects the Knowledge Objects and Learning Objectives.

Volume (teachability)

The number of Learning Objectives has increased significantly from the previous document. Depending on the time allocation for the learning of Mathematics in schools, this may have a planning and programming effect on students and teachers. The Knowledge Objects are not consistently written from year to year. They either do not develop in complexity nor cognitive demand nor do they use consistent language across the year levels.

For example

Year 1	Year 2	Year 3	Year 4	Year 5
Reading, writing and comparing natural numbers	Reading, writing, comparing and ordering numbers of up to three digits by understanding the characteristics of the decimal number system (base, the role of zero and place value)	Reading, writing, comparing and ordering four-digit natural numbers	Decimal Number System: reading, writing, comparing and ordering natural numbers of up to five digits	Decimal Number System: reading, writing, comparing and ordering natural numbers of up to five digits

There are also examples of lack of development of cognitive demand, in the one word statements in Knowledge Objects. There is no clear progression of cognitive demand or consistency.

For example

Year 1	Year 2	Year 3	Year 4	Year 5
Number line	Integers: use, history, order and association with points on a number line	Number line		Decimal Number System: reading, writing, comparing and ordering natural numbers (of up to six digits) and decimals and representing them on a number line Converting between decimals and fractions: recognition,

				meanings, reading, representation on a number line

There is no clear distinction between the Knowledge Objects and the Learning Objects. A structure for the writing of each statement needs to be clarified as there is no specificity of what constitutes an ‘Object’ as opposed to an ‘Objective’. Similar structures are used to write both and it appears they could be used interchangeably.

For example

Grade 1	Knowledge Object	Skills
Probability and statistics	Reading simple tables and column graphs	Read data displayed on tables or simple column graphs

11. Recommendation:

- A consistent structure be developed for the Knowledge Objects and the Learning Objectives to clarify their purpose.
- Consider writing the Knowledge Objects in terms of threads and the developmental progression of cognitive demand be identified by the hierarchy of the verbs used to describe the Learning Objectives.
- Ensure clear alignment between the Knowledge Objects and Learning Objectives.

Consistency

Mathematics curriculum Knowledge Objects and Learning Objectives (that is, the knowledge and skills of Mathematics) are typically hierarchical in nature and reflect key concepts or skills that are required to be learned and the order in which they are to be learned. As noted in the previous sections, the Objects and Objectives need to be reviewed in terms of their structure and use of verbs to indicate this developmental progression/heirarchy.

12. Recommendation:

A consistent structure be developed for the Knowledge Objects and the Learning Objectives to clarify their purpose.

Measurability

Most of the Learning Objectives are written with verbs that can be observed or measured. Those Learning Objectives that need to be reviewed, as per previous advice, will also need to be able to be observed and measured.

It is not clear in the documentation what is to be measured. If the Knowledge Objects are the basis for the measurability, then the clarity of the language used is paramount to their being able to be measured.

13. Recommendation:

A clear structure be developed where Knowledge Objects are clearly written so as to be observed and measured or the Knowledge Objects become set of threads or sub-headings, and the Learning Objectives form the basis for the measurement of the curriculum.

Academic Rigour

The academic demand of the Knowledge Objects and Learning Objectives encompasses a full range of cognitive demand, but there is not a steady progress of the level of demand across the years of schooling. The demand of the Knowledge Objects and Learning Objectives is reflected by the verbs used to describe the expectation of the skills. In the previous advice, Blooms Taxonomy was used to align the verbs to the level of rigour. The conclusion was there was an inconsistent degree of the growth in rigour as a student progressed through the years. In this latest version, this is still an issue. The same verbs are used to describe the skills in Year 1 as they are in Year 5. The verb 'identify' is used to describe a skill in both Year 1 and Year 8. For example: in Year 1 Geometry, 'Identify and name tow dimensional figures…' and in Year 8 Geometry, 'Identify congruent triangles…'. It would be expected that students be able to do more that identify by the time they were in Year 8. This may be an issue of translation of the text, however, it should still be noted. The use of these lower order verbs in the Learning Objectives may cause a problem for assessment of students' performance and progress. Some of the lower order cognitive verbs lend themselves to 'closed' assessment practices, where an answer is either right or wrong, as opposed to assessment of what a student is able to do and understand.

14. Recommendation:

Develop a construct as the years progress using Blooms' Taxonomy or equivalent where the number of verbs in the 'analysing', 'evaluating' and 'creating' categories increases as the years progress. This is not to imply that those verbs, such as 'compare' and 'classify', not be included in the early years, but rather that the number of these higher order verbs increase as the years progress. It would be expected that the lower order verbs be more prevalent in the earlier years.

Balance between the strands

The number of Knowledge Objects for Numbers has a decreasing trend over the years of schooling. This is expected as the abstract nature of Mathematics develops over time and students become more confident in the use of numbers. As expected, Algebra becomes more evident as the student develops more abstract thinking as they progress through the years of schooling. The number of Geometry Knowledge Objects increases after the early years and remains fairly constant across the years. Quantities and Measurements have more Learning Objectives in the early years. Probability and Statistics is more evident in the beginning of the Middle years.

15. Recommendation:

Consider the alignment of the Knowledge Objects to the Learning Objectives.

Clarity

Although the intent of the Knowledge Objects is generally clear, they are sometimes not written succinctly to describe exactly what is intended. Often they are statements, or a single word and at other times they are more like a Learning Objective. This inconsistency in the way the Knowledge Objects are written could cause teachers to be unclear about the

expectations for teaching and learning. For example: in Grade 5, in the Numbers Knowledge Objects, there are six Knowledge Objects:

1. 'Decimal Number System: reading, writing, comparing and ordering natural numbers (of up to six digits) and decimals and representing them on a number line
2. Converting between decimals and fractions: recognition, meanings, reading, representation on a number line
3. Percentages
4. Problems: addition and subtraction of natural and decimal numbers
5. Problems: multiplication and division of natural and decimal numbers
6. Solving counting problems such as: 'if each object in collection A is combined with every object of collection B, how many groups can be formed?'

The first begins with a statement and then continues by describing the skills that are required. The second one is written as a skill. The third is a single word with no further explanation. The fourth and fifth Knowledge Objects could be combined into one and the sixth Knowledge Object is written as a skill. This confusion could be eliminated if the Knowledge Objects were written as sub-headings or threads that were consistently used for each year level. The Learning Objectives could be written in the same way as they currently are in the document. This would then group the Learning Objectives into a progression across the years and would clarify the parts of the Theme Units that are to be covered.

An example of this structure is evident tin the Australian Curriculum Mathematics:

Sub-strands
Content descriptions are grouped into sub-strands to illustrate the clarity and sequence of development of concepts through and across the year levels. They support the ability to see the connections across strands and the sequential development of concepts from Foundation to Year 10.

Table 1: Content strands and sub-strands in the Australian Curriculum: Mathematics (F–10)

Number and algebra	**Measurement and geometry**	**Statistics and probability**
Number and place value (F–8)	Using units of measurement (F–10)	Chance (1–10)
Fractions and decimals (1–6)	Shape (F–7)	Data representation and interpretation (F–10)
Real numbers (7–10)	Geometric reasoning (3–10)	N/A
Money and financial mathematics (1–10)	Location and transformation (F–7)	N/A
Patterns and algebra (F–10)	Pythagoras and trigonometry (9–10)	N/A

Linear and non-linear relationships (7–10)	N/A	N/A

16. Recommendation:

Group Knowledge Objects together to form consistent threads that carry across the years as organisers for the Learning Objectives.

Summary of Recommendations:

1. It has been noted that the recommendation previously suggested to include a process strand has not been enacted. However, the inclusion of a process strand is still recommended to ensure the parity of the BNLS with international curricula. This process strand could underpin the five BNLS themes

Numbers	Geometry	Quantities and Measurements	Algebra	Probability and Statistics
Mathematical Processes				

2. Evaluate the amount of content that has been included in the BNLS with regard to the time allocated to teaching Mathematics within a whole learning programme.

3. Replace the immeasurable verbs in the BNLS with action of performance verbs that can be observed or measured.

4. The Learning Objectives be revised to achieve a balance of verbs to demonstrate the progression of cognitive development across the years of schooling.

5. Include the Algebra Knowledge Objects into the documentation for Years 7 and 8. Consider the alignment between the number of Knowledge Objects and the number of Learning Objectives in each year.

6. The Knowledge Objects be reviewed and revised to ensure that they are clear and consistent. Consideration should be given to grouping the Knowledge Objects into threads for consistency across the years. For example: Quantities and Measurements could have threads across all the year levels under the Knowledge Objects of ‘Using measurements’, ’Time’, ‘Shapes’ and ‘Money’.

7. Align the Knowledge Objects with the Learning Objectives so there is a clear line of sight between them. Consider including more references to the use of digital technologies, especially in the themes of Quantities and Measurements and Probability and Statistics.

8. Identify the General Competencies within the curriculum document.

9. Include process strands that underpin the learning of Mathematics across Elementary and Middle Schools.

10. Ensure the wording of the introductory text reflects the Knowledge Objects and Learning Objectives.

11.

- A consistent structure be developed for the Knowledge Objects and the Learning Objectives to clarify their purpose.
- Consider writing the Knowledge Objects in terms of threads and the developmental progression of cognitive demand be identified by the hierarchy of the verbs used to describe the Learning Objectives.
- Ensure clear alignment between the Knowledge Objects and Learning Objectives.

12. A consistent structure be developed for the Knowledge Objects and the Learning Objectives to clarify their purpose.

13. A clear structure be developed where Knowledge Objects are clearly written so as to be observed and measured or the Knowledge Objects become set of threads or sub-headings, and the Learning Objectives form the basis for the measurement of the curriculum.

14. Develop a construct as the years progress using Blooms' Taxonomy or equivalent where the number of verbs in the 'analysing', 'evaluating' and 'creating' categories increases as the years progress. This is not to imply that those verbs, such as 'compare' and 'classify', not be included in the early years, but rather that the number of these higher order verbs increase as the years progress. It would be expected that the lower order verbs be more prevalent in the earlier years.

15. Consider the alignment of the Knowledge Objects to the Learning Objectives

16. Group Knowledge Objects together to form consistent threads that carry across the years as organisers for the Learning Objectives.