

# **Analysis of the Brazilian National Learning Standards for Mathematics Version 3**

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## HIGH LEVEL FEEDBACK

The focus of this review report is on the alignment between the objectives and desired outcomes outlined in the overarching Introduction to the Brazilian National Learning Standards, the rationale, aims and purposes of Mathematics as articulated in the introductory text for the learning objectives, and the Mathematics learning objectives listed for both elementary school and middle school.

The writers should be congratulated for the quality of the third version of the Mathematics learning standards.

The Introductory text to the Brazilian Learning Standards contains several commitments. These provide an expectation that the subject area learning objectives will address these and contribute to student learning based on the set of principles and the acquisition of general competencies.

While it would be unreasonable to expect Mathematics to include all that is outlined in the Introduction there are some important commitments that are particularly relevant to this subject. These commitments include access to or the development of knowledge, understandings and skills concerning:

- using technology
- learning how to learn
- developing intellectual autonomy and critical thinking, and
- the relationship between theory and practice.

In terms of the Principles outlined in the Introduction, there are also some specific principles related to political and aesthetic principles that lend themselves to Mathematics. These include:

- investigating, considering, interpreting, hypothesizing and evidence base (PP)
- developing creative potential to ask questions and solve problems (AP), and
- asking questions, solving problems and multi-literacies – mathematical language (AP).

The Introduction also outlines the general competencies students should acquire under three broad organisers: social and personal competencies (SOC), cognitive competencies (COG) and communicative competencies (COM). The general competencies that students would acquire from Mathematics, which are primarily within the cognitive and communicative competencies, include knowledge, understandings and skills concerning:

- metacognitive skills pertaining to critical, creative and reflective thinking (COG)
- intellectual curiosity (COG)
- investigating, critical analysis (COG)
- investigating causes, forming and testing hypotheses (COG)
- forming and solving problems (COG)
- producing arguments based on facts and reliable information and data to formulate, negotiate and defend ideas and standpoints
- exchanging (mathematical language) information (COM), and
- explaining using facts, information and phenomena (COM).

The general competencies are also foregrounded as being particularly useful for interdisciplinary planning at the school level.

### ***Mathematics in Elementary and Middle School (4.4)***

The introductory text for Mathematics incorporates a number of the key commitments made in the Introduction to the Brazilian National Learning Standards as expressed in the principles and general competencies.

There are various statements in the section on Mathematics in elementary school and middle school that are clearly aligned with the overall principles and general competencies. Examples of these expressions and statements include students:

- making connections between empirical observations and representations (tables, graphs etc)
- finding solutions and interpreting them according to different contextual situations
- reasoning, representing, communicating and arguing
- performing investigative activities, project development and modelling
- defining problems, gathering, analysing and representing results
- judging and interpreting arguments
- constructing surveys and many types of graphs
- using many strategies to solve problems
- generalizing, estimating and interpreting
- justifying procedures and verifying the plausibility of results
- using technology (calculators and software eg electronic spreadsheets), and
- investigating properties, making assumptions and building convincing arguments.

### *Mathematics General Competencies for Elementary and Middle School*

The table illustrating the nine subject specific competencies for Mathematics and their alignment with the general competencies is particularly helpful. This information will support teachers and other educators in applying the expected emphasis to their teaching in relation to the competencies as well as support interdisciplinary planning at the school level. It will also assist policy officers and school authorities in setting assessment tasks and in monitoring the implementation of the curriculum in classrooms.

The nine subject-specific competencies for Mathematics also align well with the principles and competencies outlined in the Introduction to the Brazilian National Learning Standards. Examples of this include students:

- developing logical reasoning, investigative drive and the capacity to form arguments
- communicating understandings through appropriate representations
- investigating, organizing and representing data
- interpreting and critically evaluating data
- using digital technologies
- formulating and solving (everyday and social) problems
- interacting with peers cooperatively
- acting individually and collectively with respect, autonomy and flexibility
- feeling confident about their own abilities in mathematics, and
- recognizing Mathematics as a living science – with new mathematical knowledge being created to solve current problems and for laying the foundation for future findings.

### **Mathematics (4.4.1)**

The introductory text for Mathematics incorporates commitments made in the Brazilian National Learning Standards.

The descriptions of the five themes used as the organizational structure for the learning objectives are clear with each providing a useful overview of the scope of content students will access in both elementary school and middle school. The reference to establishing the relationship between the subject matter across the five themes (connections) as well as within each theme is reflective of the statements made in the Introduction regarding interdisciplinarity.

The emphasis is on mathematical literacy and numerical thinking is strong as is the application of mathematical knowledge, understandings and skills for solving problems. This provides clear indicators

to teachers that learning Mathematics in elementary school and middle school in Brazil should involve an appropriate balance of knowledge, understandings and skills. Reference to using technologies such as calculators and computer software is impressive.

### *Learning Mathematics in Elementary School (4.4.1.1)*

This section provides a good summary of the knowledge and dispositions children bring to school from their prior-school learning experiences. The text is brief but powerful in terms of informing teachers that planning should be based on the strengths students have and that these should be utilized to support further learning.

This section also makes clear that knowledge, though important, is not sufficient. Again, while the text in this section is brief it is also powerful. Clear direction is provided regarding the balance between acquiring mathematical knowledge, understanding and skills and applying these to solve problems, “grasp meaning” and “develop ideas and ways of thinking”.

### *Skills (Learning Objectives)*

The analysis of the Introduction to the Brazilian National Learning Standards and the sections preceding the Skills (Learning Objectives) for Elementary School was undertaken to review the alignment between overall policy intentions regarding student learning in elementary school and middle school in Brazil and the extent to which the intentions are supported in student learning in Version 3 of the draft Learning Standards for Mathematics.

As indicated previously, there is close alignment in most sections. Following is an assessment of the extent to which the same intentions are identifiable within the skills (learning objectives) for elementary school (grades 1 to 5).

The first major observation is that the section appears to be inappropriately named. The information provided about Mathematics in the earlier sections and the detailed learning objectives (presented in the third column in tables for each grade) includes knowledge, understandings and skills. An assessment of the learning objectives confirms the incorporation of these with some emphasizing knowledge and understanding and others the development and application of skills.

A more appropriate term would be: “Learning Objectives” rather than “Skills (Learning Objectives)” given that in addition to skills these objectives include knowledge and understanding.

### *Skills (Learning Objectives) for Elementary School*

In relation to the individual grades, the following assessment of the learning objectives is made in relation to elementary school.

<i>Grade</i>	<i>Comment</i>
1	Both knowledge acquisition and skill development are included in the twenty-one learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understandings. For example, students estimate, compare and solve and formulate problems as well as use mathematical language and understanding to describe, recognise and name. Counting and estimating the quantity of objects up to 100 objects would be developmentally challenging for students at this grade level. Up to 20 would be more developmentally appropriate/achievable at this age level.
2	Both knowledge acquisition and skill development are included in the twenty-two learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understanding with greater emphasis given to active rather than passive learning (students are comparing, composing, solving and formulating problems [though there is no specification if these are word problems or problems involving numbers

	and symbols only, ie equations), measuring, estimating classifying and organizing using mathematical knowledge and understanding). Counting and estimating the quantity of objects up to 1,000 objects would be developmentally challenging for students at this grade level. Up to 100 would be more developmentally appropriate/achievable at this age level.
3	Both knowledge acquisition and skill development are included in the twenty-eight learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understanding. This Grade contains the greatest number of learning objectives in elementary school Mathematics and could result in the atomization of teaching/learning.
4	Both knowledge acquisition and skill development are included in the twenty-seven learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understanding with frequent references to students solving and forming problems and undertaking investigations. The use of technologies is introduced in this grade with the use of calculators (when necessary) and digital technologies (though only as an option) in the 'Geometry' theme.
5	Both knowledge acquisition and skill development are included in the twenty-four learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understanding. Reference is made to using digital technologies in both the 'Geometry' and 'Probability and Statistics' themes though as in Grade 4 this is included as an option only – “students organize data” and “students draw” ... with <u>or</u> without digital technologies (ie there is no requirement for all students to learn how to use digital technologies).

The overall assessment is that there is a balance of knowledge, understandings and skills incorporated into the learning objectives.

There is close alignment between the intentions of the BNLS and sections of this document, as outlined earlier in this report, and the learning outcomes provided for Mathematics in elementary school. While not all of the commitments are identifiable in the learning objectives, the following are addressed either directly or indirectly:

*Understandings concerning:*

- the relationship between theory and practice
- exchanging (mathematical language) information
- explaining using facts, information and phenomena
- making connections between empirical observations and representations (tables, graphs etc), and
- reasoning, representing, communicating and arguing.

*The development of competencies including:*

- using technology (calculators)
- investigating, considering, interpreting, hypothesizing and evidence base
- developing creative potential to ask questions and solve problems
- asking questions, solving problems and multi-literacies – mathematical language
- investigating causes, forming and testing hypotheses
- forming and solving problems
- defining problems, gathering, analysing and representing results
- constructing surveys and many types of graphs
- using many strategies to solve problems
- generalizing, estimating and interpreting
- communicating understandings through appropriate representations
- interpreting and critically evaluating data
- using digital technologies (as an option only), and
- formulating and solving (everyday and social) problems.

The commitments that are less identifiable in the learning outcomes provided for Mathematics in elementary school include the following:

*Understandings concerning:*

- learning how to learn
- critical analysis
- justifying procedures and verifying the plausibility of results
- investigating properties, making assumptions and building convincing arguments
- developing logical reasoning, investigative drive and the capacity to form arguments, and
- recognizing Mathematics as a living science – with new mathematical knowledge being created to solve current problems and for laying the foundation for future findings.

*The development of competencies including:*

- developing intellectual autonomy and critical thinking
- metacognitive skills pertaining to critical, creative and reflective thinking
- intellectual curiosity
- producing arguments based on facts and reliable information and data to formulate, negotiate and defend ideas and standpoints
- finding solutions and interpreting them according to different contextual situations
- performing investigative activities, project development and modelling
- judging and interpreting arguments
- interacting with peers cooperatively
- acting individually and collectively with respect, autonomy and flexibility, and
- feeling confident about their own abilities in Mathematics.

### *Learning Mathematics in Middle School (4.4.1.2)*

The text in this section is very brief. While there are some important points made (ie recognising the knowledge, understanding and skills that students bring with them from learning Mathematics in elementary school; “grasping meaning” as a fundamental outcome; the place of the history of Mathematics) this section lacks clear insight regarding where emphasis is to be applied, the relative weightings of the five themes and the intended high level student learning outcomes for Number, Algebra, Geometry, Quantities and Measurements and Probability and Statistics.

### *Skills (Learning Objectives) for Middle School*

In relation to the individual grades, the following assessment of the learning objectives is made in relation to middle school.

<i>Grade</i>	<i>Comment</i>
6	Both knowledge acquisition and skill development are included in the twenty-one learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understanding. While the use of digital technologies (software) is listed as an option rather than a requirement in the themes of ‘Geometry’ and ‘Quantities and Measurements’, the use of electronic spreadsheets is included in the ‘Probability and Statistics’ theme. With the exception of calculators, this is the first Grade in which all students are required to demonstrate learning through the use of electronic spreadsheets.
7	Both knowledge acquisition and skill development are included in the thirty-one learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understanding. The term “understanding” is used in five of the learning objectives and should be replaced with an appropriate descriptor that clarifies how students will <u>demonstrate</u> their understanding (eg describes or identifies or demonstrates etc). The use of software in ‘Geometry’ continues as only an option in this Grade. There is a

	considerable difference in the number of learning objectives in this Grade compared to the preceding and following grades (from 21 in Grade 6 up to 31 in Grade 7 and then down to 22 in Grade 8) which could result in the atomization of teaching/learning in Grade 7.
8	Both knowledge acquisition and skill development are included in the twenty-two learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understanding. “Understanding” is used in two of the learning objectives and should be replaced with an appropriate descriptor that clarifies how students will <u>demonstrate</u> their understanding (eg describes or identifies or demonstrates etc). As with Grade 7 the use of software in ‘Geometry’ is only an option in Grade 8.
9	Both knowledge acquisition and skill development are included in the twenty-four learning objectives for this grade. Most of the objectives incorporate the acquisition of skills using acquired knowledge and understanding. “Understanding” is used in three of the learning objectives and should be replaced with an appropriate descriptor that clarifies how students will <u>demonstrate</u> their understanding (eg describes or identifies or demonstrates etc). In addition, the term “Find” is used in the first learning objective and should be replaced with an appropriate descriptor that clarifies the strategy or strategies that students should use to reach the conclusion (eg Identifies through investigation, concludes following experimentation etc). The use of software in ‘Geometry’ is a requirement in this Grade however the use of digital technologies such as a calculator is a “preferred” tool rather than its use being a requirement in ‘Number’.

The overall assessment is that there is a balance of knowledge, understandings and skills incorporated into the middle school learning objectives. Two areas that require a review are the use of technologies (particularly computer software) and the extent to which students develop mathematical understanding and its application to real world contexts through word problems.

In the case of technologies, reference is made to the use of calculators in ‘Number’ from elementary school through middle school. However, the use of computer software across the five themes is variable. There is potential for inequities to occur with some students acquiring skills, knowledge and understanding in the use of software in Mathematics from an early grade with others only accessing software from Grade 7 (in ‘Probability and Statistics’ but not ‘Geometry’ until Grade 9).

While there are frequent references to students solving and formulating problems in the middle school learning objectives for Mathematics, it is unclear if students will be engaging with Mathematics in context (word problems) or solely through numbers and symbols (number sentences/equations). Though it might be assumed that students would be engaging with word problems, the learning objectives should be reviewed to determine which will include word problems and which will be limited to numbers and symbols and insert the appropriate text to clarify the nature of the problems.

There is close alignment between the intentions of the BNLS and sections of this document, as outlined earlier in this report, and learning outcomes provided for Mathematics in middle school. While not all of the commitments are identifiable in the learning objectives for Mathematics in middle school, the following are addressed either directly or indirectly:

*Understandings concerning:*

- the relationship between theory and practice
- exchanging (mathematical language) information
- explaining using facts, information and phenomena
- making connections between empirical observations and representations (tables, graphs etc)
- reasoning, representing, communicating and arguing
- asking questions, solving problems and using mathematical language
- forming and solving problems
- finding solutions and interpreting them according to different contextual situations
- performing investigative activities, project development and modelling
- constructing surveys and many types of graphs, and

- recognizing Mathematics as a living science – with new mathematical knowledge being created to solve current problems and for laying the foundation for future findings.

*The development of competencies including:*

- using technology
- investigating, considering, interpreting, hypothesizing and evidence base
- developing creative potential to ask questions and solve problems
- critical analysis
- investigating causes, forming and testing hypotheses
- producing arguments based on facts and reliable information and data to formulate, negotiate and defend ideas and standpoints
- defining problems, gathering, analysing and representing results
- using many strategies to solve problems
- generalizing, estimating and interpreting
- justifying procedures and verifying the plausibility of results
- using technology (calculators and software eg electronic spreadsheets)
- communicating understandings through appropriate representations
- investigating, organizing and representing data
- interpreting and critically evaluating data
- using digital technologies, and
- formulating and solving (everyday and social) problems.

The commitments that are less identifiable in the learning outcomes provided for Mathematics in middle school include the following:

*Understandings concerning:*

- learning how to learn
- investigating properties, making assumptions and building convincing arguments, and
- developing logical reasoning, investigative drive and the capacity to form arguments.

*The development of competencies including:*

- developing intellectual autonomy and critical thinking
- metacognitive skills pertaining to critical, creative and reflective thinking
- intellectual curiosity
- judging and interpreting arguments
- interacting with peers cooperatively
- acting individually and collectively with respect, autonomy and flexibility, and
- feeling confident about their own abilities in Mathematics.

## **SUMMARY and RECOMMENDATIONS**

The introductory text and specific section describing Mathematics align well with the Introduction to the Brazilian National Learning Standards.

Principles and competencies outlined in the Introduction to the Brazilian National Learning Standards, which provide clear commitments regarding student learning in elementary school and middle school in Brazil, are identifiable in the learning objectives. These are complemented with the subject-specific competencies identified for Mathematics.

While there are some principles and competencies that are not well-represented in the learning objectives, the overall impression is that students will be active learners in Mathematics and acquire important knowledge, understandings and skills.

The following key recommendations have been drawn from the review of the descriptions of Mathematics and the specific learning objectives for elementary school and middle school.



## Recommendations

1. The sections outlining the learning objectives for both elementary school and middle school should be renamed, replacing ‘Skills (Learning Objectives)’ with ‘Learning Objectives’.
2. The writers review the many references to “solve and formulate problems” to determine where qualification is needed (ie when are word problems introduced and when are students engaging with problems that are solely number sentences etc?) and inserting the appropriate text eg “solving and formulating word problems involving...”.
3. The writers review the use of technology (specifically in relation to the use of computer software across the five themes) and make appropriate changes to requirements to avoid inequities regarding students’ access to digital technologies.
4. The writers replace the terms “Understands” and “Finds” in learning objectives in Grades 7-9 with appropriate descriptors to illustrate how students will demonstrate understanding or that they have found (reached an understanding).
5. The elementary school and middle school learning objectives be reviewed by the writers to identify opportunities to incorporate (or if necessary include additional learning objectives) where students develop knowledge and understandings, skills, values and attitudes described in the overarching Introduction to the Brazilian National Learning Standards and in the introductory sections for Mathematics concerning:
  - learning how to learn
  - investigating properties, making assumptions and building convincing arguments
  - developing logical reasoning, investigative drive and the capacity to form arguments
  - developing intellectual autonomy and critical thinking
  - metacognitive skills pertaining to critical, creative and reflective thinking
  - intellectual curiosity
  - judging and interpreting arguments
  - interacting with peers cooperatively
  - acting individually and collectively with respect, autonomy and flexibility, and
  - feeling confident about their own abilities in Mathematics.